PRN:		Course Code :	MTS 304

Third Year B.Sc End Semester Examination Winter 2022

Integral Calculus

Backlog

Day and Date: Wed, 14-12-2022 Max Marks: **100**

Time : 10:30 AM - 1:30 PM Duration: **03** hrs

Instructions:

1) All questions are compulsory.

2) **Sub question 'a)' is compulsory, Solve any two from the remaining**

3) Figures to the right indicate full marks

4) **Use of Non- Programmable calculator is allowed**

		Marks	Bloom's Level	CO
Q.1	Attempt the following			
a)	Evaluate $\int \sin^3 x \cos^4 x\,dx$.	6	L3	CO1
b)	Find the reduction formula for $\int \sin^n x\,dx$, where n is a positive integer.	7	L2	CO1
c)	Evaluate $\int \cot^5 x\,dx$	7	L3	CO1
d)	Find the reduction formula for $\int e^{ax} \sin^n bx\,dx$.	7	L2	CO1
Q.2	Attempt the following			
a)	If $f:[a,b]\to\mathbb{R}$ be a bounded function and P be any partition on $[a,b]$ then prove that $m(b-a)\le L(P,f)\le U(P,F)\le M(b-a)$, where m,M are greatest lower bound and least upper bound respectively of $f(x)$ in $[a,b]$.	6	L2	CO2
b)	Prove that every constant function is $R-$integrable.	7	L2	CO2
c)	Show that $f(x)=x$ is $R-$integrable on $[0,1]$.	7	L2	CO2
d)	State and prove Darboux's theorem.	7	L2	CO2
Q.3	Attempt the following			
a)	Test the convergence of $\int_0^\infty \frac{\sin x}{x^4}dx$.	6	L3	CO3
b)	Test the convergence of	7	L3	CO3

Backlog

i) $\int_0^\infty \frac{x}{(1+x)^3}dx$, ii) $\int_1^\infty \frac{1}{x^{1/3}(1+x)^{1/2}}dx$

c) If $f(x)$ and $\phi(x)$ are both positive, bounded and integrable function on (a,∞), for all $x \geq a$ and $\lim_{x\to\infty} \frac{f(x)}{\phi(x)} = l$, where l is a finite non-zero number then show that the integral $\int_a^\infty f(x)dx$ and $\int_a^\infty \phi(x)dx$ behave alike. 7 L2 CO3

d) Test the convergence of $\int_a^b \frac{1}{(b-x)^p}dx$. 7 L3 CO3

Q.4 Attempt the following

a) Evaluate $\int_0^1 (\log x)^4 dx$. 6 L3 CO4

b) Evaluate $\int_5^9 \sqrt{(9-x)(x-5)}\ dx$. 7 L3 CO4

c) Evaluate $\int_0^1 x^6\left(1-x^2\right)^{\frac{1}{2}} dx$ 7 L3 CO4

d) Evaluate $\int_0^{\frac{\pi}{6}} \cos^6 3x \cdot \sin^2 6x\, dx$. 7 L3 CO4

Q.5 Attempt the following

a) Evaluate $\int_0^{\pi}\int_0^{a(1+\cos\theta)} r\ dr\, d\theta$ 6 L3 CO5

b) Evaluate $\int_0^1\int_{x^2}^{x}\left(x^2y+y^2x\right)dy\, dx$ 7 L3 CO5

c) Find the area of the circle $x^2+y^2=a^2$ by using double integral. 7 L3 CO5

d) Change the order of integration and evaluate $\int_0^5\int_{2-x}^{2+x} dy\, dx$. 7 L3 CO5

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Backlog